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for the trisection of an angle $\hat{A}BC$. Take any point D on AB produced and make $BC = BD$; through C draw a parallel to BA and on it lay off a length $CP = CD$; then $\hat{A}BP$ is approximately $\frac{1}{3}\hat{A}BC$. Show that this construction is rendered very much more accurate by laying off on CP a distance $CQ = 2BD$ and taking in place of P , the point R which divides PQ in the ratio 5 : 4. Also show that the error does not exceed $10''$ for angles less than 45° .

2973. Proposed by N. K. CHAFFEE, Rutland, Vt.

A straight brass bar 800 feet long expands 8 inches. The ends are fixed, so that it is distorted. If the new shape is that of an arc of a circle of which the original bar is the chord, how far above the center of the bar in its original position will the center be in the new position?

It is instructive to guess at this distance before attempting to solve the problem.

2974. Proposed by the late L. G. WELD.

A man standing on a straight railway track watches a train starting from a station half a mile distant and notices that it approaches at such speed that each puff of the exhaust is heard at the same instant that the next succeeding puff is seen. How long will it be before the train reaches him, the drive wheels of the locomotive being sixteen feet in circumference?

2975. Proposed by the late ARTEMAS MARTIN.

What must be the form of the mold-board of a plow, considered as a warped surface, so that it will run most easily through the ground?

2976. Proposed by NATHAN ALTSHILLER-COURT, University of Oklahoma.

The base of a variable triangle is fixed, the opposite vertex describing a straight line. Find the locus of the symmedian point, the locus of the center of the nine-point circle, and the envelope of the Euler line.

SOLUTIONS.

2825 [1920, 186]. Proposed by the late L. G. WELD.

A ball, having a coefficient of resilience α , strikes a rigid plane surface, inclined at an angle θ from the horizontal, after falling through a height h . What is the distance from the first to the second point of impact with the plane?

SOLUTION BY WILLIAM HOOVER, Columbus, Ohio, and H. P. MANNING, Providence, R. I.

In this problem, the forces are impulsive. Let $u = \sqrt{2gh}$ be the velocity of the center of the ball when it impinges upon the plane for the first time at O ; let the straight line through O directly down the plane be the axis of x , and the normal to the plane at O the axis of y ; let v_x, v_y be the x and y components of the space velocity immediately after the first impact; and let X, Y be the corresponding reactions between the ball and plane, the mutual friction being such that the ball does not slip. Let m be the mass, and a the radius of the ball, and let k be its radius of gyration about a diameter.

First suppose $\alpha = 0$. Then, resolving parallel to the x and y axes, we have

$$m(v_x - u \sin \theta) = -X, \quad m(v_y + u \cos \theta) = Y;$$

and, taking moments about the center of the ball, and putting ω = the angular velocity, we have also

$$mk^2\omega = Xa.$$

For the condition of no slipping, we have $v_x = a\omega$, and for $\alpha = 0, v_y = 0$. These equations give

$$X = \frac{mk^2u \sin \theta}{k^2 + a^2}, \quad Y = mu \cos \theta. \quad (1)$$

Let v_x', v_y' be the velocities corresponding to v_x, v_y in the second case, when resilience is taken into account; then for X, Y we write $(1 + \alpha)X, (1 + \alpha)Y$, and the equations for linear motion become

$$m(v_x' - u \sin \theta) = -(1 + \alpha)X, \quad m(v_y' + u \cos \theta) = (1 + \alpha)Y.$$

These combined with (1) give

$$v_x' = u \sin \theta \frac{a^2 - \alpha k^2}{a^2 + k^2}, \quad v_y' = + u \alpha \cos \theta. \quad (2)$$

The equations of motion are $\ddot{x} = g \sin \theta$, $\ddot{y} = -g \cos \theta$, or $\dot{x} = gt \sin \theta + c_1$, $\dot{y} = -gt \cos \theta + c_2$. When $t = 0$, $\dot{x} = v_x'$ and $\dot{y} = v_y'$. Hence determining c_1 and c_2 from (2), we get

$$\dot{x} = \sin \theta \left(gt + u \frac{a^2 - \alpha k^2}{a^2 + k^2} \right), \quad \dot{y} = \cos \theta (u \alpha - gt).$$

Integrating again, with the conditions $x = 0$ and $y = a$ when $t = 0$,

$$x = \sin \theta \left(\frac{1}{2}gt^2 + tu \frac{a^2 - \alpha k^2}{a^2 + k^2} \right), \quad y = a + \cos \theta (u \alpha t - \frac{1}{2}gt^2).$$

The ball hits the plane a second time when $y = a$, i.e. when $t = 2u\alpha/g$. The corresponding value of x becomes, after simplification,

$$\frac{2a^2\alpha(1 + \alpha)u^2}{g(a^2 + k^2)} \sin \theta.$$

Also solved by H. L. OLSON, ARTHUR PELLETIER, and J. B. REYNOLDS.

2860 [1920, 428]. Proposed by E. O. BROWN, Chicago, Ill.

A frustum of a right circular cone has a volume v . The lateral area added to the lesser base is a sum which is a minimum. Determine the dimensions of the frustum in terms of v .

SOLUTION BY H. S. UHLER, Yale University.

Let h , r , R , and s denote respectively the altitude, the radius of the smaller base, the radius of the larger base, and the slant height of the frustum. Also let σ symbolize the sum in question. Then

$$\sigma = \pi(Rs + rs + r^2), \quad (1)$$

$$v = \frac{1}{3}\pi h(R^2 + Rr + r^2), \quad (2)$$

$$s = + [h^2 + (R - r)^2]^{1/2}. \quad (3)$$

By virtue of equations (2) and (3), σ may be considered as a function of the two independent variables R and r , hence the necessary conditions for a minimum of σ are

$$\frac{\partial \sigma}{\partial R} = 0, \quad \frac{\partial \sigma}{\partial r} = 0.$$

Differentiating equations (1), (2), and (3) with respect to R and combining the results we obtain

$$\frac{\partial \sigma}{\partial R} = \frac{\pi}{s} \left\{ s^2 + (R + r) \left[R - r - \frac{9v^2(2R + r)}{\pi^2(R^2 + Rr + r^2)^3} \right] \right\} = 0,$$

whence

$$\frac{9v^2}{\pi^2(R^2 + Rr + r^2)^3} = \frac{2(R - r)}{R + 2r}. \quad (4)$$

In like manner we find

$$\frac{\partial \sigma}{\partial r} = \frac{\pi r}{s} \left[2s - 2(R - r) - \frac{9v^2(2R + r)}{\pi^2(R^2 + Rr + r^2)^3} \right] = 0. \quad (5)$$

The ratio $r/R \equiv \rho$ may be obtained in the following manner. First substitute the equal of the left member of equation (4) in the third or fractional term of equation (5) to get

$$s = \frac{3(R^2 - r^2)}{R + 2r}. \quad (6)$$

Then square the last equation, eliminate v^2 by again using relation (4), and suppress the com-